

Academic Scholarship 2013

## Mathematics

Paper 1

Time Allowed: $\mathbf{1}$ hour and $\mathbf{3 0}$ minutes

Calculators may NOT be used for this paper

## Instructions to candidates:

- You may answer the questions in any order.
- Remember to show your working and clearly show the method you are using.

1. Work out the following:
a) $\quad 5.9 \times 4.32$
b) $\sqrt{\frac{32000}{0.002}}$
c) $\frac{2+\frac{1}{2}}{3-\frac{1}{3}}$
2. If $\frac{37.9 \times 4.2}{8.24+0.53}=18.15051311$

Without doing lots of lengthy calculations, write down the value of:
a) $\frac{0.0379 \times 0.042}{8.24+0.53}$
b) $\frac{37.9 \times 4.2}{824+53}$
c) $\frac{379 \times 2.1}{0.824+0.053}$
3. a) Showing your working, approximate the numbers in this calculation to estimate the value of $\sqrt{\frac{4.87 \times 319}{9.9-9.89}}$, giving your answer to 1 sig fig.
b) Work out $9 \times(8 \times 7 \times(6-5) \times 4-3+2 \times 1)$
4. Expand out if necessary and simplify the following expressions:
a) $\quad 12-4(x+y)$
b) $2 y(y+3)-5 y(1-y)$
c) $\quad 2 x^{2} y \times(3 x)^{3} y$
5. Solve the following equations:
a) $\quad 4(x+2)-3(5-2 x)=7$
b) $\frac{3 x+1}{2}-\frac{2 x+3}{3}=-5$
c) $\quad(x+3)^{2}=100$
6. The PIN number of a debit card consists of a 4 digit number, eg. 7429 in which each digit can be $0,1,2,3,4,5,6,7,8$, or 9 . Hence there are 10 different digits and therefore 10000 possible PIN numbers.
a) How many PIN numbers start with a 5?
b) How many PIN number have no digits greater than 2?
c) How many PIN numbers have no digits greater than 5?
7. Ten students sit an exam which has a maximum score of 100. The average of the scores of 10 students in a class is 92 . What is the lowest mark that a student in the class could have scored? Give your reasoning clearly.
8. Solve the following pair of simultaneous equations;
i) $3 x-5 y=7$
$5 x+2 y=22$
ii) Using the solutions that you have obtained to this pair of equations work out all the solutions to the simultaneous equations below:
a) $3 x+5 y=7$
b) $\quad 3 x^{2}-5 y^{2}=7$
c) $3 y-5 x=7 x y$
$5 x-2 y=22$
$5 x^{2}+2 y^{2}=22$
$5 y+2 x=22 x y$
9. By considering the last digits of powers of 2 and 5 , find the last digit of $2^{34}+5^{67}$.
10. Deborah drew a sequence of square grid patterns with diamonds in some of the small squares as shown.
$S_{1}$

| $\diamond$ | 0 |
| :--- | :--- |
| $\diamond$ | 0 |

$S_{2}$

|  | 0 | 0 |
| :--- | :--- | :--- |
| 0 | 0 | 0 |
| 0 | 0 |  |

$S_{3}$

|  |  | $\diamond$ | 0 |
| :--- | :--- | :--- | :--- |
|  | $\diamond$ | 0 | 0 |
| $\diamond$ | $\diamond$ | 0 |  |
| $\diamond$ | $\diamond$ |  |  |

a) Draw $S_{4}$.
b) How many diamonds are there in $S 5$ ?
c) How many diamonds are there in $S_{n}$ ? (give your answer in terms of $n$ )
d) How many small squares in Sn do not contain diamonds?
11. a) On the same set of axes sketch the graph of $y=x^{2}-3$ and $y=x-3$.
$|x|$ means the modulus (or size) of $x$, taking no account of the sign.
So $|3.5|=3.5 \quad$ and $\quad|-4.8|=4.8$
b) On separate axes, sketch the graphs of:
i) $\quad y=|x|$
ii) $\quad y=|x-3|$
iii) $\quad y=\left|x^{2}-3\right|$

Label the coordinates of any points where the graphs touch the axes.

